The 2nd Industrial Conference for Applied Research (24 – 25, October 2023)

ICAR 2023
Special Issue

Iraqi Journal of Industrial Research, Vol. 11, No. 2 (2024)

Iraqi Journal of Industrial Research (IJOIR)

Journal homepage: http://ijoir.gov.iq

Exploring Fatigue Crack Initiation in Helical Gears: Impact of Carburization and Frictional Influences

Abdullah Dhayea Assi

Department of Mechanical Engineering, College of Engineering, University of Baghdad, Iraq

Article information

Article history:
Received: October, 30, 2023
Accepted: January, 08, 2024
Available online: October, 20, 2024

Keywords:
Heat treatment,
Friction effects,
Fatigue criterion

Corresponding Author:
Abdullah Dhayea Assi
drabdullahdhayea@uobaghdad.edu.iq

DOI:
https://doi.org/10.53523/ijoirVol11I2ID379

Abstract

A computational model is presented to analyze the fatigue crack initiation period in helical gears, incorporating considerations for heat treatment via carburization and friction effects. The aim is to determine the number of stress cycles necessary for the initiation and growth of initial cracks, while investigating the impact of dynamic behavior. This study employs a dynamic gear model with two degrees of freedom in torsion, developed using MATLAB, and incorporates the Papadopoulos criterion for fatigue analysis. The computational results are benchmarked against the strain-life method. The findings underscore the significant influence of the friction coefficient between surfaces, heat treatment, and dynamic loads on the growth of initial fatigue cracks. For instance, with a friction coefficient (μ) of 0.5, the initial crack forms on the tooth surface (y=0) in both methods, with the (ε-N) method suggesting 28-65 cycles and the Papadopoulos criterion indicating 101-156 cycles. The latter accounts for the greater influence of residual stresses. Additionally, untreated gears exhibit a minimum number of loading cycles for initial crack appearance at 1.07×10^4 cycles. In contrast, carburized gears demonstrate an extended lifespan, requiring 1.45×10^5 loading cycles for crack initiation in our example. This emphasizes the efficacy of carburization in enhancing gear durability.

1. Introduction

The mechanical behavior of the various elements of machines, such as gears, rollers, couplings ... etc, depends on the mutual effect between the conditions of the applied load and the characteristics of the surfaces, where the surfaces, in particular, subject to the movement of a vehicle from sliding and rolling in the conditions of repeated loading, as in the case of gears, to the growth of fatigue, which is considered a type of damage associated with structural changes of the material. In general, the growth of fatigue depends on many factors, some of which are related to the conditions of lubrication, the roughness of the surface, the properties of the material and the methods of its treatment, while others are related to the values of stresses and elastic reactions formed within the layers of the material resulting from the conditions of loading. In contrast, the number of total loading cycles of fatigue growth can be determined (as a set of cycles number Ni) necessary for the growth or formation of the fatigue crack initiation within the layers of the material and the number of cycles (Np) required for the fatigue crack propagation from its initial length to its critical length.

Due to the advantages of meshing performance and long service life, helical gears have been widely applied in mechanical power transmission systems. Under high-speed and heavy-duty conditions, helical gears might incur contact fatigue failures, such as pitting and spalling due to insufficient tooth flank bearing capacity, offset load, or tooth profile error, etc. [1–3], as well as incurring tooth breaking due to insufficient bending strength **[4].** As such, it was necessary to combine machining processes and the heat treatment process of gears to complete basic research, including residual stress measurement, failure morphology analysis, to provide theoretical and experimental bases for the failure analysis of contact fatigue tests and fatigue life prediction. The existing research on gear residual stress mainly focuses on the effect of machining or the heat treatment process (Carburization) on the residual stress and Friction Effects of the tooth flank and the effect of residual stress on fatigue.

Various studies have delved into the intricate relationship between manufacturing processes and the residual stress characteristics of gears, offering valuable insights into enhancing their mechanical properties. Rego et al. [5] explored the impact of shot peening, grinding, and heat treatment on the residual stress of 16MnCr5 spur gears during machining. Conrado et al. [6–9] investigated the correlation between tooth root residual stress and the bending fatigue strength of spur gears fabricated from diverse materials, subjected to carburizing and nitriding processes. Lingamanaik and Chen [10] conducted carburization treatments on SAE 5120 automotive gears, revealing that extended carburization durations led to increased residual compressive stress and a deeper hardened layer. Savaria et al. [11, 12] focused on the influence of various degrees of induction quenching on the residual stress of AMS6414 spur gears, highlighting the pivotal role of residual compressive stress in improving bending fatigue strength. Mallipeddi et al. [13] scrutinized the evolution of residual stress in 16CrMn5 spur gear tooth roots during the running-in period. Liu et al. [14] explored the impact of residual stress on the tooth flank of helical gears in wind turbines, observing a correlation between decreasing compressive residual stress and the formation of internal cracks farther from the surface. Fukumasu et al. [15] measured compressive residual stress near the surface regions of 17NiCrMo helical gears. Lv et al. [16] discovered the release of residual compressive stress on the surface of W6Mo5Cr4V2 gears after contact fatigue. Pariente et al. [17] determined the residual stress value of 16CrNi4 gears after contact fatigue testing, observing the release of initial tooth root residual stress near the surface zone. Kattoura et al. [18] additionally identified that high twin content and dislocation densities could impede the formation and development of surface fatigue cracks. These collective findings contribute to a comprehensive understanding of the interplay between manufacturing processes and the residual stress characteristics critical to the performance of gears.

Depending on the literature reviews, the period of formation of the crack initiation is considered one of the most important stages of collapse with fatigue [19-22], and this is based on the fact that the time of propagated of any crack initiation, formed within the layers of the material for a mechanical element subject to repeated loading, is very fast compared to the time of its propagation, and as a result, it is difficult to control In the propagated of the incision or even prevent its propagated in the conditions of repeated loading. For this reason, the attention is often directed towards studying the stage of the initial crack, with the aim of calculating the number of loading cycles required for its growth, or researching methods of preventing its occurrence. In contrast, the emotion method (strain - the number of cycles), which is denoted by an abbreviation (ε-N), is considered one of the most important and most used methods to study the period of formation of the initial crack as this method is based on analyzing contact stresses, taking into account the specific fatigue parameters for each substance, (ε'_f): Index of ductility, σ'_f: index of fatigue strength in bending, E: modulus of elasticity ... etc.). Depending on this method, the increase in the amount of plastic strain ($\Delta\varepsilon_p$) and elastic strain ($\Delta\varepsilon_\theta$) and the number of cycles are related to the following relationship:

$$\frac{\Delta\varepsilon}{2}=\frac{\Delta\varepsilon_\theta}{2}+\frac{\Delta\varepsilon_p}{2}=\frac{\sigma'_f}{E}(2N_i)^b+\varepsilon'_f(2N_i)^c \qquad (1)$$

where b, c: exponent is related to ductility and fatigue strength, respectively and taking into account the influence of the mean stresses $\boldsymbol{\sigma_m}$, the previous relationship is as follows:

$$\frac{\Delta\varepsilon}{2}=\frac{\Delta\varepsilon_\theta}{2}+\frac{\Delta\varepsilon_p}{2}=\frac{(\sigma'_f-\sigma_m)}{E}(2N_i)^b+\varepsilon'_f(2N_i)^c \qquad (2)$$

Despite the importance of the method (strain - number of cycles) (ε-N) in analyzing the period of formation of the fatigue crack initial, but it assumes that the material is homogeneous without any defects, in the crystal pattern (imperfection), nor does it take into account the effect of roughness and the type of heat treatment (for the residuals stresses) . In addition, we can use both the shear stresses (τ_{xy}), the shear stresses (τ_{max}) and the maximum principal stresses in analyzing this method, and as a result we obtain different values for the number of cycles needed to be the initiation crack in each case, in return, all studies in the scientific references do not take into consideration the effect of dynamic loading conditions on the growth of the initial crack within the layers of the gears, where the study is limited to using a simplified model of gears as two equal disks, and consequently, the resulting loading conditions are considered static conditions.

For this purpose, the main aim of this research is to present an arithmetic model that helps in calculating the number of cycles needed to form the crack initiation on the teeth of the helical gears, based on the Papadopoulos criterion for fatigue prediction, and using a dynamic model of the two-degree gears, freedom to twist, taking into account the effect of both the dynamic behavior of the gears, the type of heat treatment (effect of residual stresses) and the effect of friction between the contact surfaces.

3. Experimental Works

Using the MATLAB program, it has been developed dynamic model of gear with two freedom degrees in torsion, in order to determine and calculate the dynamic load and the maximum pressure value at each contact point on the teeth of the gears, by analyzing the dynamic movement equation step by step with time. Depending on the value of the resulting maximum pressure, the Normal contact stresses and their distribution within the layers of matter are calculated. Results obtained using Papadopoulos criterion will be compared to results obtained using the (ε-N) method.

3.1. Modeling the Dynamic Behavior of Helical Gears

Gears with helical teeth are widely used in the transmission of motion, because of its qualities and functional characteristics that distinguish it from gears with straight teeth. For example, these gears are less affected by manufacturing errors as deflection and shape errors. This is due to the slope of the contact lines over the entire width of the tooth (d) with respect to the axis of the tooth at an angle called the helix angle (β). Which also causes a difference in the value of both the real pitch (p_t) and the actual pitch (normal pitch) (p_n) between the teeth (Figure-1). It is also more suitable for high speeds and withstands large loads due to the presence of more number of teeth in the interlock phase simultaneously, In addition to the low noise and vibration level. Despite these important advantages, the conditions for manufacturing, modeling and studying the conditions of dynamic interlock are more difficult than their straight-tooth counterparts.

In our present research, we will rely on a dynamic model with two degrees of freedom to twist (θ_1 for the gear & θ_2 for the pinion) shown in Figure (2) [23].

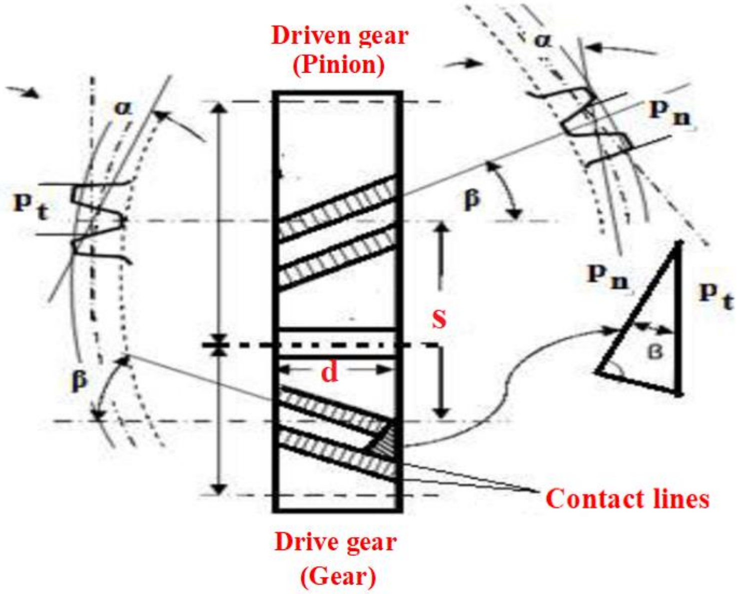

Figure (1): Conditions for Contact between Helical Gear Teeth.

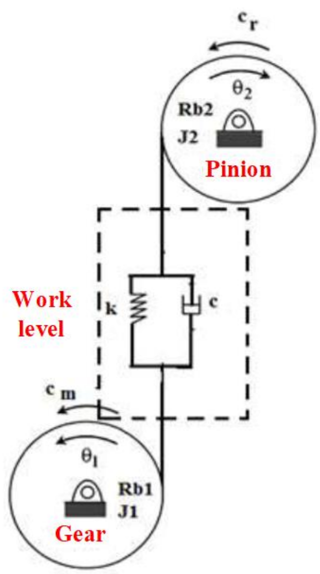

Figure (2): A dynamic model of gears with two degree of freedom [23].

The gear and the pinion are described by its polar breakout torque (J), the bending radius (Rb) and the applied torque value. Also the torque on the gear (torque motor) (Cm) and the torque on the pinion) (Cr). On the other hand, the conditions of contact between the teeth of the gears are described within the working level with the interlock hardness (K) and the damping constant (C). Relying on the dynamic torque theory, the equation of movement for both the gear and pinion are written according to the following differential formula [23]:

$$J_1\left[\dot{\Omega}_1+\ddot{\theta}_1\right]=-\left\{C.\left[Rb_1.\dot{\theta}_1+Rb_2.\dot{\theta}_2\right]+K.\left[Rb_1.\theta_1+Rb_2.\theta_2\right]\right\}.\cos^2\beta.Rb_2+C_m \quad (3)$$

$$J_2\left[\dot{\Omega}+\ddot{\theta}_2\right]=-\left\{C.\left[Rb_1.\dot{\theta}_1+Rb_2.\dot{\theta}_2\right]+K.\left[Rb_1.\theta_1+Rb_2.\theta_2\right]\right\}\cos^2\beta.Rb_2+C_r \quad (4)$$

Since the angular velocity for both the gear Ω1 and the pinion Ω2 is constant and therefore Ω1=Ω2=0, we can express the previous two equations by the following formula:

$$\begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix}\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}+.\left\{C.\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}+K.\cos^2\beta.\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}\right\}.\begin{bmatrix} Rb_1^2 & Rb_1Rb_2 \\ Rb_1Rb_2 & Rb_2^2 \end{bmatrix}=\begin{bmatrix} c_r \\ c_m \end{bmatrix} \quad (5)$$

By multiplying the first line of the equation by Rb_1J_2 and the second line by Rb_1J_2 and dividing the equation by $(Rb_2)^2J_1 + (Rb_1)^2J_2$ we get the following differential equation:

$$m_{tot}.\ddot{x}+c.\dot{x}+k.x.\cos^2\beta=F_s \quad (6)$$

β: the helix angle (the angle of inclination of the contact lines between the teeth of the gears within the working plane) and equal in the case of studying the straight gears of zero ($\beta = 0$).

$$m_{tot}=\frac{J_2J_2}{J_2Rb_2^2+J_2Rb_1^2} \quad (7)$$

m_{tot}: the total mass equivalent of the gears and given by the following relationship:

C, k: equivalent hardness interlock constant and damping of the dynamic pattern respectively calculated according to the ISO 6336 system [24].

x: wave of degrees of freedom in the twisting of the gear and pinion, and given according to the relationship:

$$x=Rb_1\theta_1+Rb_2\theta_2 \quad (8)$$

Fs: static load affecting gears, calculated according to the following relationship:

$$F_s=\frac{c_m}{Rb_1}=\frac{c_1}{Rb_2} \quad (9)$$

The solution to the previous motion equation will be step-by-step over time, using the Newmark method [25], which is considered one of the most used methods of analysis in finding solutions to dynamic problems, and analyzing the movement equations for the transmission terms with gears. During the analysis of the motion equation, it must be ensured that there is no negative load on the teeth of the gears. Through each step of time, the tangent lines that tilt at the helix angle (β) are shifted within the work plane and therefore the tangent point M will move with time across the contact lines and so the engineering features of the gears (distribution of load - distribution of pressure - width of the contact area ... .) It will be variable due to the difference in the location of the contact point on the tooth profile. For the purpose of the analysis, we used 48 interlocking intervals with the aim of obtaining a stable system of interlocking gear, the total number of increments in time equal to 2048. Since the point of contact between the teeth of the gear and the pinion moves along the profile of the tooth through time, for this, the profile of the teeth of the gears will be described in relation to (t/T_m) where t: time, T_m: the interlock period. During the calculation, the amount of time increase Δt is calculated according to the relationship ($\Delta t = t_m / tt$), where tt: the number of time increments during a single interlock period, which is calculated by the number of total increments in time divided by the number of interlock periods. That interlock period (Tm) is calculated based on the angular velocity of the gear Ω1 and the number of its teeth Z1 according to the following relationship:

$$T_m = \frac{2\pi}{Z_1 . \Omega_1} \quad (10)$$

After calculating the dynamic load F at each point of contact M over the teeth of the gears (noting that the distance between the points of contact for two consecutive gears = p.cosα where p: pith. α: pressure angle) in relation to the width of the tooth (d), the distribution of the normal contact pressure P(x) on the width of the contact area H is calculated using the Hertz theory according to the following [22] Figure (3):

$$P(X) = \frac{2.\pi}{\alpha H(M)} . \frac{F(M)}{d} \sqrt{1 - \frac{X}{\alpha H(M)}} \quad (11)$$

While the tangential pressure q (x) is calculated using the Coulomb theory according to the following relationship:

$$q(x) = \mu . p(x) \quad (12)$$

The coefficient of friction μ will be calculated using the velocity of slipping ($v_s = u_1 - u_2$) and the rotational speed ($v_r = u_1 + u_2$) and the kinematic viscosity η_0 according to the following, [26]:

$$\mu = 0.0127 \log \left[\frac{29.66 \frac{F(M_U}{d}}{\eta_0 v_s v_r^2} \right] \quad (12)$$

u_1, u_2: the circumferential velocity of the gear and pinion, respectively.

After determining the studied point, the total stress field is the sum of the stresses due to the normal contact load σ^ni,j(x,y), and the stresses due to the tangential load σ^ti,j(x,y). These normal contact and tangential stresses are calculated within the conditions of plane distortions, [27-28].

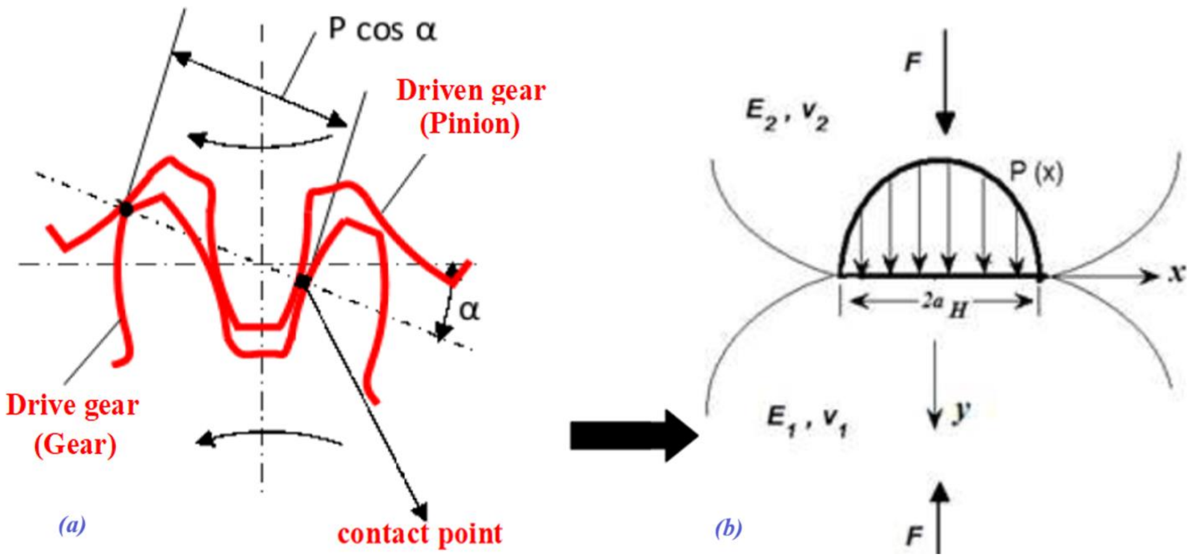

Figure (3): Modeling the distribution of normal and tangential pressure at the contact point E, ν: modulus of elasticity and Poisson ratio respectively. (1) Drive gear (Gear) & (2) Driven gear (pinion).

On the other hand, the gears used in the transmission of movement are often heat treated with the aim of increasing the transferred capacity, and because of this treatment arises in the layers of the material negative residual stresses, where these stresses lead to an important role in increasing the resistance of gears to fatigue, for this purpose the residual stresses are distributed $\sigma r_{i,j}$ (x,y) within the matter layers will be taken into account. Thus, the total stress field at each contact point is given by the following relationship:

$$\sigma_{i,j}(x,y) = \sigma^{n}_{i,j}(x,y) + \sigma^{t}_{i,j}(x,y) + \sigma^{r}_{i,j}(x,y) \forall i, j' \in x, y, z \quad \textbf{(14)}$$

There are many experimental relationships in the scientific references to calculate the distribution of stresses, according to the change in hardness within the layers of the material, which relate to the type of heat treatments for the surfaces of gears (hardening), (nitriding) & (carburizing). In the present work, we will use the experimental relationship provided by the scientist (Tobe), to describe the distribution of the residual stresses in the case of carbonization [29], according to this relationship, the residual stresses are distributed, depending on the depth beneath the surface of the gears, relates to the values of the Vickers hardness in the surface (HVS) and the heart (HVC).

3.2. Modeling the Period of the Fatigue Crack Initiation

Recent studies of fatigue prediction rely on the use of Fatigue criterion. There are many of these standards in the scientific literature, but the most widely used are CROSSLAND, DANG VAN, SINES, and PAPADOULOS. In our study presented here, we will rely on the Papadoulos criterion, [30], which gives great compatibility with experimental results, [30]. This criterion is based on the maximum value of the second constant of the stresses wave $\sqrt{J_{2,a}}$ and the hydrostatic pressure PH$\left(p_H = \frac{1}{3}\left[\sigma_{xx} + \sigma_{yy} + \sigma_{zz}\right]\right)$, taking into account the effect of time.

Depending on this criterion, the initial fatigue on the gears teeth arises when the values of the equivalent stresses of this σ_{eqS} criterion are greater than the values of the fatigue limit in the rotating twist, otherwise the gears are safe against fatigue. Where this criterion is expressed according to the following mathematical formula:

$$\sigma_{eqs} = \max\left(\sqrt{J_{2,a}(t)}\right) + \gamma.\left[\max\left(p_H(t)\right)\right] \le \alpha \quad \textbf{(15)}$$

where, α ,γ are the parameters of the material which are calculated based on the value of fatigue limits in the alternate twist T_W and the alternate bending σ_W according to the following:

$$\gamma = 3\left(\frac{\tau_w}{\sigma_w} - \frac{1}{\sqrt{3}}\right) \qquad \alpha = \tau_w \qquad (16)$$

$$\sqrt{J_{2,a}} = \sqrt{\frac{1}{6}\left[(\sigma_{1,a} - \sigma_{2,a})^2 + (\sigma_{1,a} - \sigma_{3,a})^2 + (\sigma_{2,a} + \sigma_{3,a})^2\right]} \quad MPa \quad (17)$$

where σ_1, σ_2, σ_3 the principal stresses of the normal and tangential stress compounds. Where the amplitude stress is calculated by relying on the maximum σ_{max} and minimum σ_{min} value of the stress according to the following relationship:

$$\sigma_{i,a} = \frac{\sigma_{i,\max} - \sigma_{i,\min}}{2} \qquad i = 1,2,3 \qquad (18)$$

That the gears are often treated thermally in order to gain the properties of the resistance and therefore the limits of fatigue τ_w and σ_w will be considered variable within the layers of the material and calculated depending on the change of the hardness of the material HV (y) according to the following relationships [31]:

$$\sigma_w = \frac{\tau_w}{0.577} = 1.61.\frac{HV(y)}{(1 + \frac{20.7}{HV(y)})} \qquad [\mathrm{MPa}] \qquad \text{for } \mathrm{HV(y)} \leq 340\mathrm{HV} \quad (19)$$

$$\sigma_w = \frac{\tau_w}{0.577} = \frac{1.98.HV(y) - 0.0011.HV^2(y)}{\left(1 + \frac{20.7}{HV(y)}\right)} \qquad [\mathrm{MPa}] \qquad \text{for } \mathrm{HV(y)} > 340\mathrm{HV} \quad (20)$$

Thus fatigue occurs when the value of the equivalent stresses σ_{eqS} becomes greater or equal to the value of the fatigue limit in the rotating twist t_W. When this condition is met, we can calculate the number of cycles needed to form the initial crack, Ni, based on a Woher curve of fatigue that can be expressed depending on the fatigue properties of the material (which are considered as special features for each substance related to the type of heat treatment), shear stresses (ta) and amplitude stresses. (σ_a) according to the following [32]:

$$\sigma_a = \sigma'_f \left(2N_i\right)^b \qquad (21)$$

$$\tau_a = \tau'_f \left(2N_i\right)^b \qquad (22)$$

where $\sigma_f^{'}$, $\tau_f^{'}$ the fatigue strength coefficients in torsion and bending respectively [32], b = power is associated with fatigue strength.

By applying these relationships in the cases of torsion and bending or tension - compression the Papadopoulos criterion, we will be able to calculate the number of cycles needed to be the initial crack according to the following:

In the case of torsion, normal stresses are non-existent and therefore:

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = 0 \Rightarrow P_H = 0 \qquad (23)$$

The Papadopoulos criterion will have the following shape:

$$\max\sqrt{J_{2,a}(t)}+\gamma.0=\alpha \Rightarrow \max\sqrt{J_{2,a}(t)}=\tau_a=\alpha \Rightarrow \alpha=\tau'_f\,(2N_i)^b \quad (24)$$

In the case of tension - Compression, with a stress ratio: -1= σmin / σmax

$$\frac{\sigma_a}{\sqrt{3}}+\gamma\frac{\sigma_a}{3}=\alpha \Rightarrow \frac{\sigma'_f\,(2N_i)^b}{\sqrt{3}}+\gamma\frac{\sigma'_f\,(2N_i)^b}{3}=\tau'_f\,(2N_i)^b \Rightarrow \gamma=3\left[\frac{\tau'_f}{\sigma'_f}-\frac{1}{\sqrt{3}}\right] \quad (25)$$

After determining the values of each of the γ and α constant in terms of fatigue parameters of the material, the new form of Papadopoulos criterion will become according to the following:

$$\max(\sqrt{J_{2,a}(t)})+3\left[\frac{\tau'_f}{\sigma'_f}-3\right][\max(P_H(t))]\leq \tau'_f\,(2N_i)^b \quad (26)$$

As a result, the number of cycles required to form the initial crack (Ni) is calculated from the relationship:

$$N_i=0.5\left[\frac{\max(\sqrt{J_{2,a}(t)}+3\left[\frac{\tau'_f}{\sigma'_f}-\frac{1}{\sqrt{3}}\right][\max(P_H(t))]}{\tau'_f}\right]^{\frac{1}{b}} \quad (27)$$

3.3. Metal Used

Gears made of 18CrMo4 steel and carburizing treatment. The chemical composition of this material and the parameters of fatigue are shown in Table (1) and taken from reference [33]. The engineering and functional data for helical gears used in this study are shown in a Table (2).

Table (1): The chemical composition and the fatigue parameters of 18CrMo4 steel.

The Chemical Composition		**The Parameters of Fatigue**	
C	0.203%	σ'_f	2244 Mpa
Si	0.27%	τ'_f	1353 Mpa
Mn	0.84%	ε'_f	0.27 m/m
p	0.022%	b	0.097-
S	0.042%	c	0.398-
Cr	1.02%	n	0.244
Ni	0.02%	Surface hardness(HV_S)	710HV
Mo	0.2%	the heart hardness (HV_c)	375HV

Table (2): The engineering and functional data for helical gears used in this study.

Dates	**Values**
The number of teeth of the gear and the pinion	Z1=18,Z2=16
Width of the gear and the pinion (mm)	b1=16, b2=16
Model (mm)	3.5
Angle of pressure (degrees)	20
Helix angle (°)	15
Engine torque (N.m)	250
gear rotating speed (rad / esc)	Ω_1=100
Lubrication properties: ISOVG 150 mineral oil	η_0=1.56 × 10^{-2} pa.s

4. Results and Discussion

When examining the fatigue phenomenon of gears, it is important to know the value and distribution of maximum pressure at contact points along the effective profile of the tooth, because the value of stresses (responsible for the occurrence of this phenomenon) and their distribution within the layers of the material largely depend on the values of maximum pressure, where the most dangerous points for the growth Fatigue is the point that has a higher pressure value compared to the low value point (ISO 2006) [24].

The results obtained by the distribution of pressure on the teeth of the gears are shown in Figure (4), where we can notice that the distribution of the maximum pressure is oblique along the width of the spiral tooth (due to the slope of the contact lines at the angle of the helix (β) where the maximum value is placed at the starting point of the contact line (in the case of the gears with teeth for a straight line, the distribution of pressure is straight, as β = 0). On the other hand, on the profile of the tooth, the distribution of pressure is approximately uniform, with maximum values of pressure placed in the vicinity of the pitch point and the points of interlock (point A), with higher values in the starting points of interlock. Conversely, in the interlock end points (point B) the maximum pressure values are low compared to a value in the interlock starting points (point A). Thus we can say that the points of interlock in the vicinity of the pitch point and the beginning of interlock are the most dangerous for the emergence of fatigue.

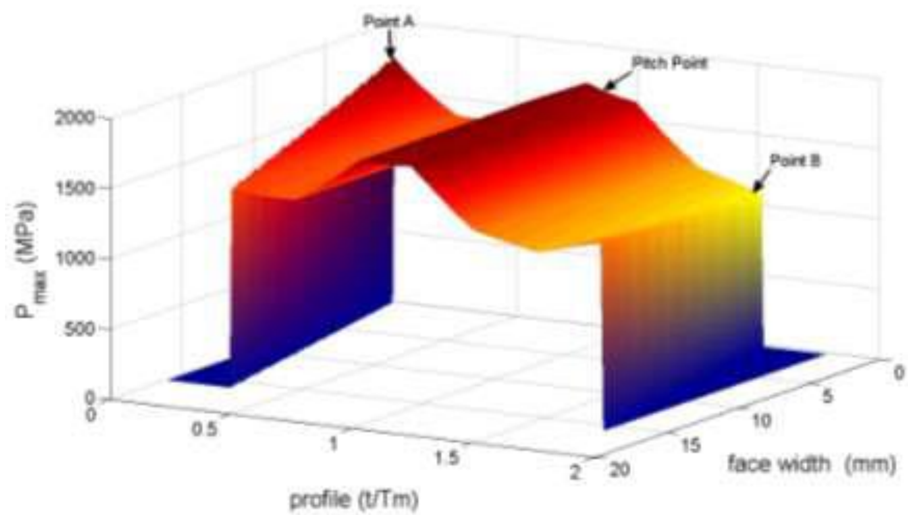

Figure (4): 3D modeling using the MATLAB program to distribute the maximum pressure Pmax over the helical tooth profile represented by the dimensionless time (t / Tm) and the tooth width depending on the new-mark method and the gears data in Table (2).

On the other hand, the tooth profile of the gears is subjected to a complex movement of slippage and rotation, Figure (5), according to the following:

- At the pitch point (dedendum), the slip is non-moving and the movement is only rotational (due to the lack of the speed of the slip at this point where the speed of the gear and pinion is equal).
- In the area between the starting points of interlock and the pitch point (dedendum) the sliding movement and rotation are both. With a sliding movement of negative value and opposite to the rotational movement of positive values.
- In the area between the pitch point (dedendum) and the interlocking end points (addendum), both the rotational and sliding movements are present, but with positive values for the sliding movement as the rotation direction corresponds to the direction of slipping because the sliding and rotational speeds have the same signal.

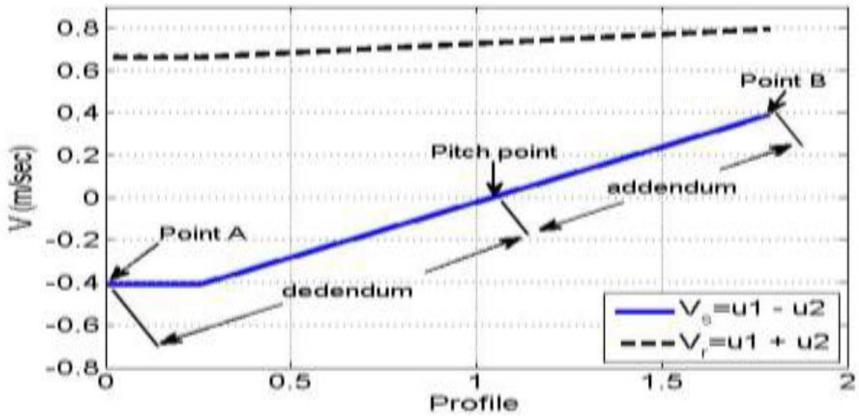

Figure (5): The sliding speed Vs and the rotational speed Vr are distributed along the effective profile of a gear depending on the gear data in Table (2).

Depending on the results shown in Figures (4 & 5), we can verify that the interlock points located between the pitch point and the starting interlock points are the most dangerous points for the growth of fatigue on the teeth of helical gears due to the high values of pressure on the one hand, and due to the negative sliding opposite the direction of movement Rotational, on the other hand, with a greater probability of the onset of fatigue at the starting points of interlock (higher pressure values and greater negative sliding) and these results correspond to experimental observations of the growth of fatigue on the teeth of helical gears as shown in Figure (6).

Based on the above, we can conclude that the starting point of interlock (point A) is the most dangerous point for the onset of fatigue on the tooth profile. Let's take this point that has a maximum pressure value of 1750MPa and study the possibility of fatigue exposure based on the Papadopoulos criterion. Initially, it is necessary to know the distribution of Vickers hardness and the value of the residual stresses, as well as the distribution of fatigue limits in the rotating twist τw and the alternating bending σw depending on the depth below the surface of the tooth. These results for modeling the distribution of hardness and residual stresses using experimental relationships of the scientist Tobe [19] are shown in Figure (7). Where we can notice that the hardness of the material (18CrMo4) treated with carbonization is greater in the area adjacent to the surface of the tooth. We can also note that the residual stresses are negative ones, and therefore they contribute to increasing the resistance of the gears teeth of fatigue, as they reach a maximum value near the surface, and then decrease until they are absent at a distance of 0.7mm from the surface shape (7a). The change in the hardness of the gears according to the depth of the surface will lead to a change in the distribution of fatigue limits, Figure (7b). As fatigue limits reach their maximum values near the surface to decrease as we move away from the surface.

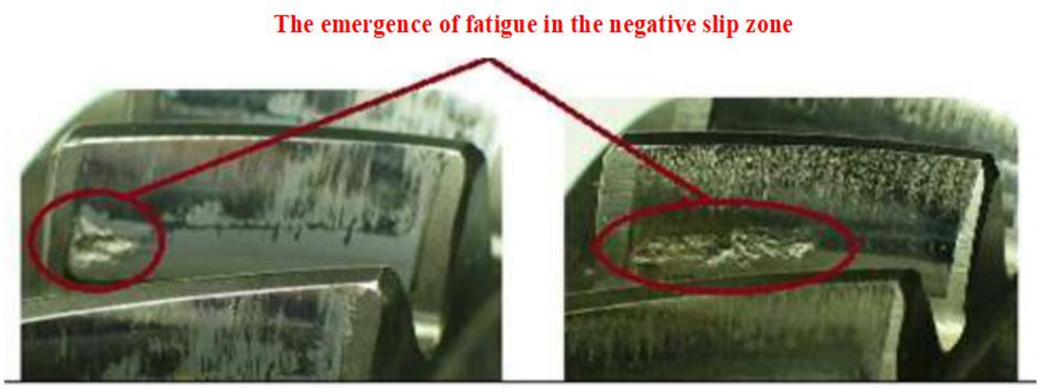

Figure (6): Experimental results for zones of fatigue on helical gears teeth.

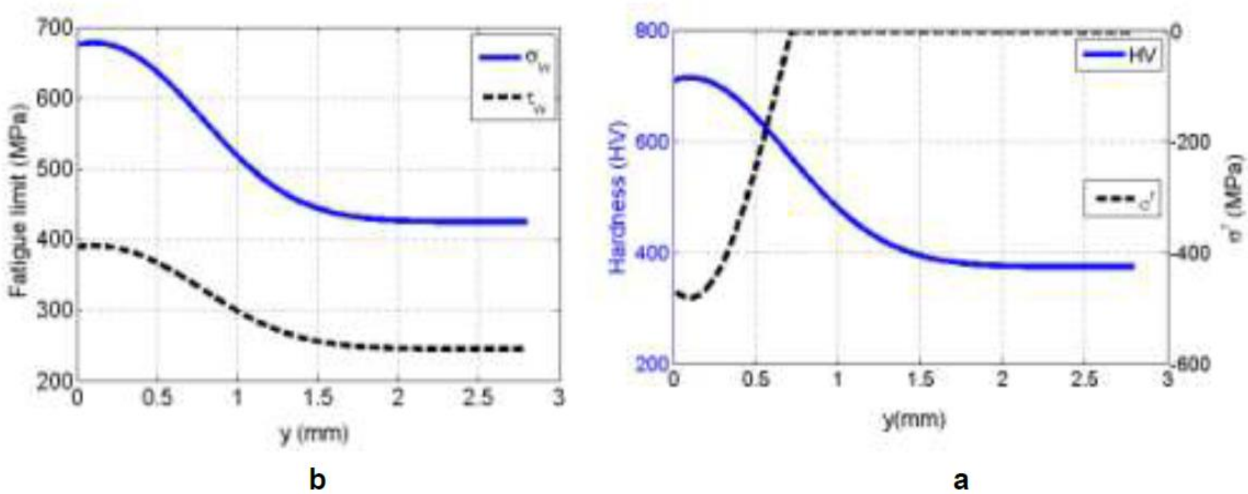

Figure (7): (a) The distribution of the gear material Hardness and the remaining stresses depending on the depth below the tooth surface (y), and (b) The distribution of the fatigue limits alternating torsion and alternating bending depending on the depth below the tooth surface (y).

Figure (8a) shows the distribution of the equivalent stresses σ_{eqS} of the Papadopoulos criterion within the material layers of the gears teeth. The value of these equivalent stresses, with or without taking into account the residual stresses σr, is greater than the value of the fatigue limit in the rotating twist, and as a result there is a risk to start the fatigue cracks initiation. The fatigue cracks initiation is located within the layers of matter is the point corresponding to the maximum value of equivalent stresses, and this point is located 0.185mm from the surface in our example.

On the other hand, the residual stresses decrease the value and distribute the equivalent stresses within the layers of the material up to a distance of 0.7mm from the surface (the depth at which you own the value), and therefore they reduce the speed of fatigue initiation due to the decrease in the value of the equivalent stresses, the effect of these residual stresses on the number of loading cycles necessary for the fatigue crack initiation Ni is shown in Figure (8b), where we can notice that the number of the minimum loading cycles for the emergence of the initial crack in the case of non-treatment gears is 1.07×10^4 cycle, while when the gears are heat treated with carbonization in our example, the number of loading cycles is 1.45×10^5 cycle. As a result, the residual stresses (resulting from heat treatment) contribute to increasing the number of loading cycles necessary for the emergence of the initial crack and thus increasing the working life of the gears as a result of the rescue of the value of the equivalent stresses formed within the layers of the material. This effect of the residual stresses is neglected using a (ε-N) method.

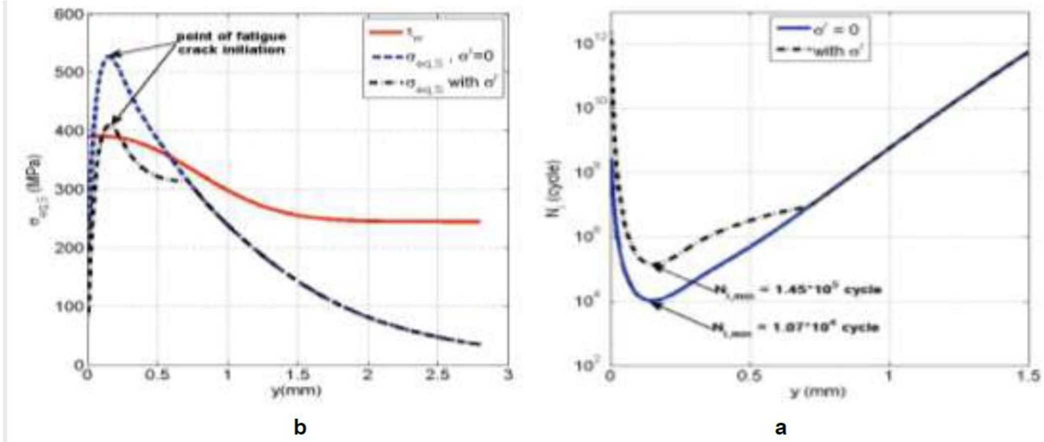

Figure (8): (a) The equivalent stresses σ_{eqS} to the Papadopoulos criterion depending on the depth below the tooth surface (y), and (b) The number of loading cycles N_i required for the formation of the fatigue crack initiation at the starting point of interlock (point A).

5. Results Comparison

Now compare the results using the Papadopoulos criterion, with those obtained using the shear stresses (τ_{xy}) in the (ε-N) method for the starting points of interlock (point A), with taking into account the effect of friction between the gear teeth. The obtained results are shown in Figure (9), with values of the friction coefficient ranging between (μ= 0-0.5) taking into account the effect of the mean stresses (σm) in the (ε-N) method, where we can observe the following:

- The method (ε-N) gives approximate results with the Papadopoulos criterion when neglecting the effect of the residual stresses. In both cases, With values of the friction coefficient equal to zero (μ=0), the initial crack of fatigue within the layers of the material arises at a distance from the surface with a number of cycles ranging from 1×10^4 to 8.2×10^3 cycle according to the method (ε-N) with and without taking into consideration the effect of the mean stresses (σm), respectively, to the number of cycles 2.2×10^4 cycle (according to the fatigue criterion and neglecting the effect of the residual stresses ($\sigma_r = 0$)). However, this difference is increased when the effect of the residual stresses is taken into consideration, as the number of loading cycles is 5×10^5 cycle according to the Papadopoulos criterion.
- When the friction values increase between the gear teeth, the region of the fatigue crack initiation within the layers of the material shifts toward the surface, and the higher the friction values, the more the region of the crack will approach to the surface. The number of loading cycles required for the formation or initial this crack decreases, due to the high values of equivalent stresses resulting from the increase in the values of tangential stresses that relate to the values of friction between the surfaces. For example, in the case of (μ = 0.5), the initial crack is formed on the surface of the tooth (y = 0) in both methods, with the number of cycles (28-65) cycle according to the (ε-N) method, bringing the number of cycles to (101-156) cycle using Papadopoulos criterion. Where the greater value indicates the effect of the residual stresses. The large decrease in the number of cycles as a result of increased friction can be explained by increasing the value of equivalent stresses σ_{eqS} resulting from the increase in the values of tangential stresses and consequently decreasing the value of Ni and in this case fatigue becomes low fatigue number of cycles (Ni $<10^3$ cycle) depending on the Wohler curve of fatigue. This result is consistent with the experimental results described in references [34, 35]. As a result, fatigue measurements give more accurate results in calculating the number of loading cycles required for the initial fatigue crack, because they take into account the effect of heat treatments for gears, this effect which is neglected using the (ε-N) method.

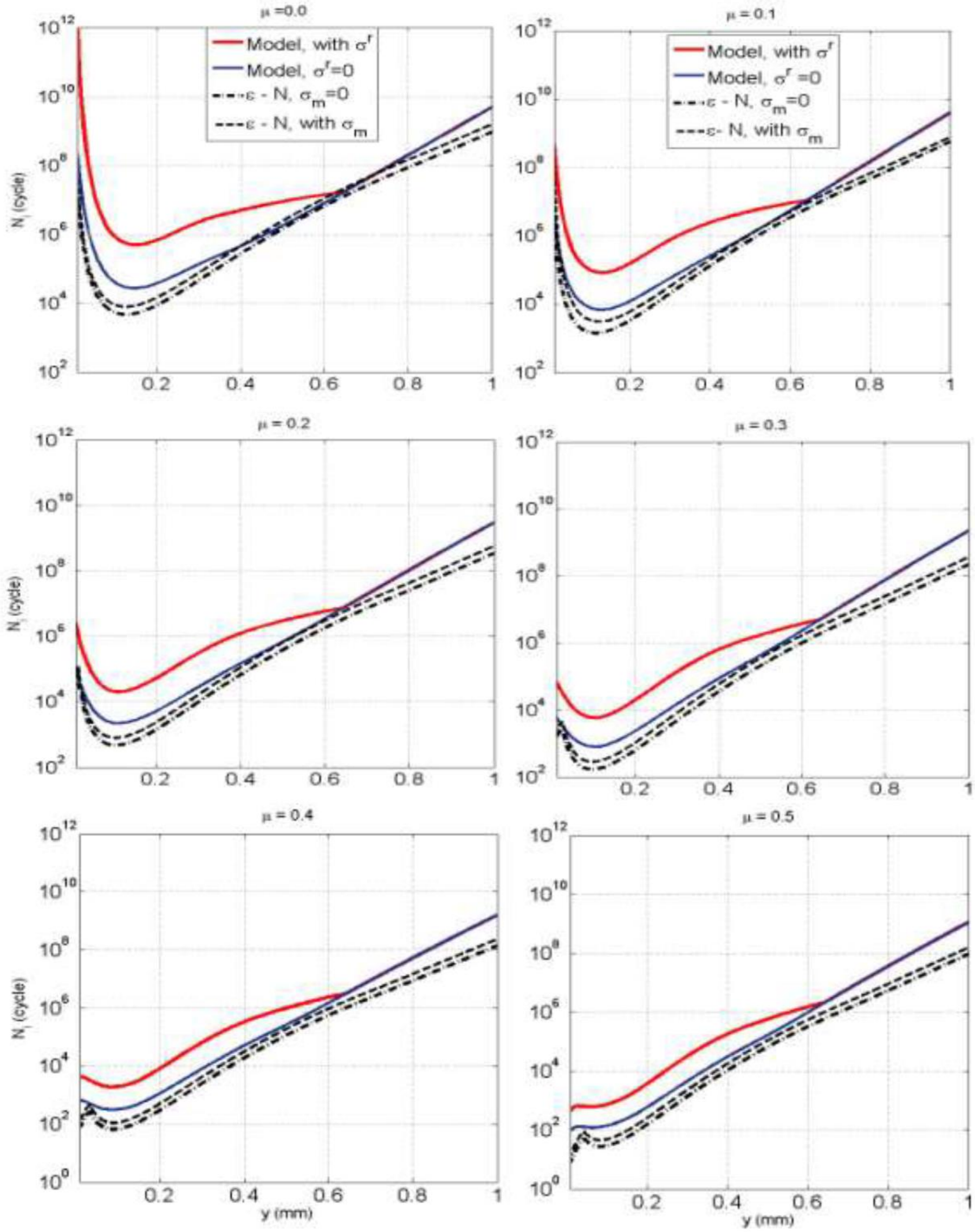

Figure (9): A comparison between the application of the Papadopoulos fatigue criterion and the (ε-N) method to calculate the number of loading cycles N_i required to produce the initial crack of fatigue under the influence of friction depending on the depth below the tooth surface at the starting point of the interlock (point A), (P_{max} = 1750 MPa).

The distribution of normal pressure on the teeth of gears depends on the number of cycles of gears, at low rotational speeds, its distribution is almost uniform along the profile, and the behavior of gears is considered static behavior, but with increasing values of rotational speed, its distribution becomes more random, and the behavior of gears it is a dynamic behavior. These notes are shown in Figure (10), where we can observe the effect of the rotational velocity of the gear Ω_1 on the distribution of the normal contact pressure on the tooth profile. Where we can notice that the starting point for interlock (point A) has a greater pressure value compared to all contact points on the tooth profile.

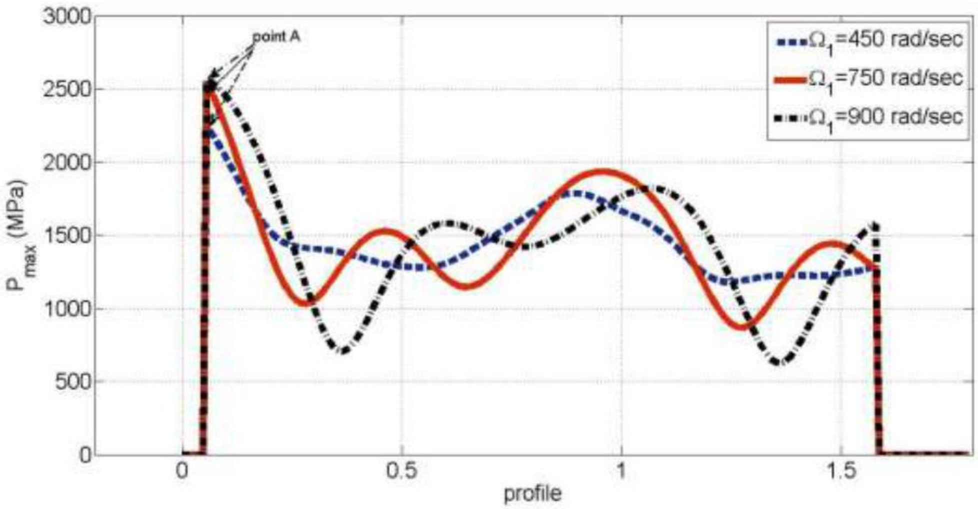

Figure (10): The effect of the angular velocity of the (drive gear) gear rotation on the distribution of maximum pressure on the helical tooth profile.

Thus, the difference in rotational speed does not change the location of the dangerous point for the initial of fatigue over the profile of the tooth, but it changes the value of the maximum pressure at this point, and therefore from the number of loading cycles necessary for the initial of fatigue. These observations are shown in Figure (11), which shows the effect of rotational speed of the gear on the maximum values of the equivalent stresses σ_{eqS}, and as a result, its effect on the number of loading cycles required for the emergence of the initial crack in relation to the point of interlock starting (Point A) and the pitch point. Where we can notice the following:

The starting point of interlock: the equivalent stresses with respect to the fatigue limit in the alternating twist are always greater than one for all rotational speeds, and therefore fatigue will arise at this point regardless of the rotational speed value. On the other hand, each rotational velocity gives a different value to the risk of fatigue initial, and as a result, the number of cycles Ni for the emergence of the initiation crack (the speed of the formation of the initiation crack) is different.

Pitch point: The initial crack is formed at a speed lower than the starting point of the interlock (fewer cycles) due to the low values of equivalent stresses, noting that some rotational speed does not give the risk of the initial crack forming at this point Ω_1= (200, 225, 875, 900, 925, 950, 975) rad/sec, and that's because:

$$\left(\frac{\sigma_{eqs}}{\tau_w} > 1\right) \quad \text{...(28)}$$

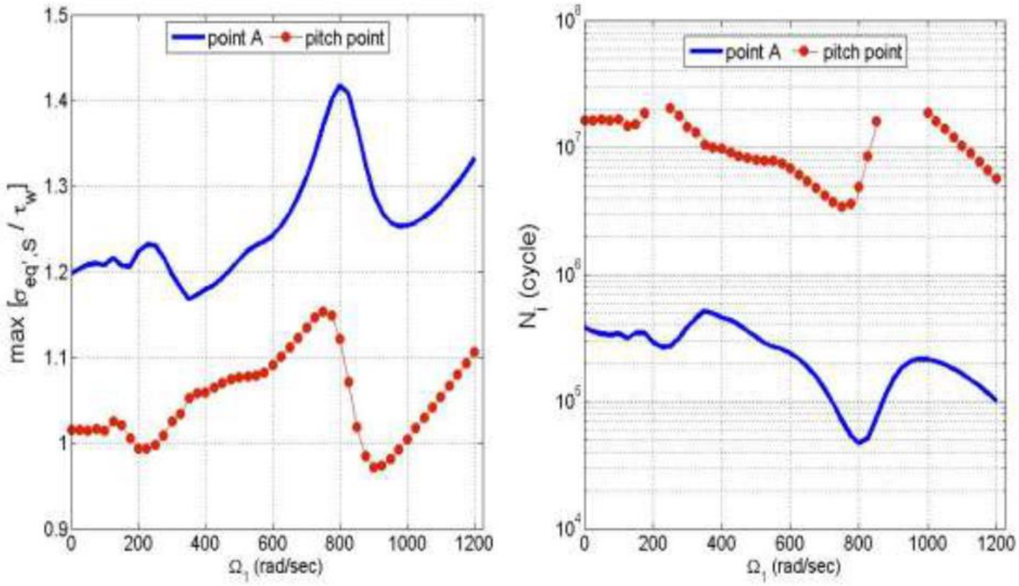

Figure (11): Effect of the angular velocity of the driving gear rotation on the value of the equivalent stresses in relation to the fatigue limit in the rotating twist and the number of loading cycles required for the emergence of the initial fatigue crack.

6. Conclusions and Recommendations

In this paper, we presented a dynamic modeling of the period of the fatigue crack initiation on the teeth of helical gears, by predicting it on the one hand and calculating the number of loading cycles necessary for the emergence of the initial crack on the other. Based on the Papadopoulos criterion for fatigue prediction and fatigue parameters for the material, taking into account the effect of friction (surface condition) and type of heat treatment. The dynamic behavior of the gears was determined by analyzing the motion equation step-by-step over time for the two-degree model of freedom of twist. The results obtained were compared with (strain - number of cycles) (ε-N) method, It is widely used in scientific references. Where we can conclude the following:

1. The starting points of interlock are the most dangerous points for teeth of helical gears for the emergence of fatigue. This is due to the higher values of maximum pressure compared to the rest of the contact points on the gear teeth. As high values of maximum pressure will lead to the emergence of the fatigue crack initiation (decrease of loading cycles Ni).
2. The dynamic behavior of helical gears affects the speed of fatigue formation due to its effect on pressure values and as a result of its effect on the values of stresses formed within the layers of the material.
3. The Gears are often heat treated to give them the characteristics of resistance (for fatigue, for example). As a result, the (ε-N) method cannot be used, as it eliminates the effect of residual stresses resulting from these treatments. Accordingly, the use of fatigue criterion is better for predicting the working life of heat treated gears.
4. The increase in the friction values between the surfaces of the gears leads to an increase in the values of the formed stresses, and consequently an increase in the speed of fatigue formation (reducing the number of loading cycles required for the emergence of the initial crack) and shifting the area of the origin of the initial crack towards the surface of the gears. Accordingly, a good choice of the type of lubricant between the surfaces of the gears, and attention to the degree of termination of their surfaces, will contribute to reducing the friction values between the surfaces.

The remaining stresses resulting from heat treatments, formed within the layers of the material, are negative ones, and therefore they increase the fatigue resistance because they decrease the value of the stresses formed within the layers of the material. For this, it is necessary to research more in choosing the type of heat treatment (carbonization, intruding, hardening ... etc) appropriate for the gears according to their working conditions.

Conflict of Interest: The authors declare that there are no conflicts of interest associated with this research project. We have no financial or personal relationships that could potentially bias our work or influence the interpretation of the results.

References

[1] J. Kattelus, J. Miettinen and A. Lehtovaara, "Detection of gear pitting failure progression with on-line particle monitoring. Tribol," *Int.*, vol. 118, pp. 458–464, 2018.
[2] H. Ma, Z. Li and M. Feng, "Time-varying mesh stiffness calculation of spur gears with spalling defect," *Eng. Fail. Anal.*, vol .66, pp. 166–176, 2016.
[3] J. Zhang, W. Li, H. Wang, Q. Song and L. Lu, "A comparison of the effects of traditional shot peening and micro-shot peening on the scuffing resistance of carburized and quenched gear steel," *Wear*, vol. 368, pp. 253–257, 2016.
[4] M. Hein, T. Tobie and K. Stahl, "Parameter study on the calculated risk of tooth flank fracture of case hardened gears". In Proceedings of the JSME International Conference on Motion and Power Tran., Kyoto, Japan, 1–3 March 2017.
[5] R. Rego, C. Löpenhaus, J. Gomes and F. Klocke, "Residual stress interaction on gear manufacturing", *J. Mater. Process. Technol*, vol. 252, pp. 249–258, 2018.
[6] E. Conrado, C. Gorla, P. Davoli and M. Boniardi, "A comparison of bending fatigue strength of carburized and nitrided gears for industrial applications," *Eng. Fail. Anal*, vol. 78, pp. 41–54, 2017.
[7] C. Gorla, F. Rosa, F. Concli and H. Albertini, "Bending fatigue strength of innovative gear materials for wind turbines gearboxes: Effect of surface coatings. In Proceedings of the ASME International Mechanical Engineering Congress and Exposition," Houston, TX, USA, 9–15 November; pp. 3141–3147, 2012.
[8] C. Gorla, E. Conrado, F. Rosa and F. Concli, "Contact and bending fatigue behaviour of austempered ductile iron gears," *Proc. Inst. Mech. Eng. Part C J. Mech. Eng. Sci.*, vol. 232, pp. 998–1008, 2018.
[9] C. Gorla, F. Rosa, E. Conrado and F. Concli, "Bending fatigue strength of case carburized and nitrided gear steels for aeronautical applications," *Int. J. Appl. Eng. Res.*, vol. 12, pp. 11306–11322, 2017.
[10] A. D. Assi, "A new Track of Fatigue crack growth in Aluminum Alloy (2219) under Cyclic Stresses", *Journal of Techniques*, ISSN: 2708-8383, Vol. 2, No. 4, December 31, pp. 33-46, 2020.
[11] V. Savaria, H. Monajati, and P. Bocher, "Measurement and correction of residual stress gradients in aeronautical gears after various induction surface hardening treatments," *J. Mater. Process. Technol*, vol. 220, pp. 113–123, 2015.
[12] V. Savaria, F. Bridier and P. Bocher, "Predicting the effects of material properties gradient and residual stresses on the bending fatigue strength of induction hardened aeronautical gears," *Int. J. Fatigue*, vol. 85, pp. 70–84, 2016.
[13] D. Mallipeddi, M. Norell, M. Sosa and L. Nyborg, "Influence of running-in on surface characteristics of efficiency tested ground gears," *Tribol. Int.*, vol. 115, pp. 45–58, 2017.
[14] H. Liu, H. Liu, P. Bocher and C. Zhu, "Effects of the case hardening properties on the contact fatigue of a wind turbine gear pair," *Int. J. Mech. Sci.*, vol. 141, pp. 520–527, 2018.
[15] N. K. Fukumasu, G. A. Machado, R. M. Souza and I. F. Machado, "Stress analysis to improve pitting resistance in gear teeth," *Procedia CIRP*, vol. 45, pp. 255–258, 2016.
[16] Y. Lv, L. Lei and L. Sun, "Effect of microshot peened treatment on the fatigue behavior of laser-melted W6Mo5Cr4V2 steel gear," *Int. J. Fatigue*, vol. 98, pp. 121–130, 2017.
[17] A. D. Assi & R. M. Hamed," Fatigue Limit Prediction Based on Hardness for Both Steel and Aluminum Alloys", *IOP: Materials Science and Engineering*, vol. 1105, No.012044, pp. 1-11, 2021.
[18] M. Kattoura, A. Telang, S. Mannava and D. Qian,; Vasudevan, V.K. Effect of ultrasonic nanocrystal surface modification on residual stress, microstructure and fatigue behavior of ATI 718Plus alloy. *Mater. Sci. Eng. A*, vol. 711, pp. 364–377, 2018.
[19] C. Francesca and M. Carlo, "Crack propagation behavior in planet gears" *Procedia Structural Integrity*," vol. 2, pp. 3610 – 3616, 2016.
[20] A. D. Assi and B. M. Abod, "Fatigue Life Prediction of AA6063-T6 under Erosion Condition", Materials Science Forum Submitted: 2020-06-19, ISSN: 1662-9752, Vol. 1021, pp 87-96, 2020.
[21] A. D. Assi1 & S. H. Omran, "Effect of Corrosion on the Fatigue Strength of the Medium Carbon Steel (CK 50) after a Shot Peening Technique", *Journal of Mechanical Engineering Research and Developments*, vol. 43, No. 6 pp. 64-73, Published Year 2020.

[22] M. Sramel and J. Flasker, “Computational approach to contact fatigue damage initiation analysis of gear teeth flanks,” International Journal of Vanced Manufacturing Technology , vol. 31, pp. 1066-1075, 2007.
[23] P. Velex and, M. Ajmi “On the modeling of excitations in geared systems by transmission errors”, *Journal of Sound and Vibration*,” vol. 290, pp. 882-909, 2006.
[24] ISO 6336. Calculation of load carrying capacity of spur and helical gears part5: strength and quality of materials, 2006.
[25] T. Khabou, M. Bouchaala, N. Chaari, F. Fakhfakh and M. Haddar, “Study of a spur gear dynamic behavior in transient regime”, *Mechanical Systems and Signal Processing*, vol. 25, pp.3089-3101, 2011.
[26] N. Anderson, E. Loewethal and H. “Spur gear system efficiency at part and full load,” Technical Report 46-79, NASA Technical paper 1622-1980.
[27] H. Qiao, H. Evans and R. Snidle, “Comparison of fatigue model results for rough surface elst to hydro dynamic lubrication,” *Journal of Engineering Triboiogy, part j*, vol. 222, pp. 381-393, 2008.
[28] T. Jabbor, A. Ghazi and T. Ghazi, “Tooth stress calculation of metal spur and helical gears,” *Mechanics and Machine Theory*, vol. 92, p. 375-390, 2015.
[29] M. N. Abdulridah, A. D. Assi and H. J. Al-Alkawi, “Influence of Cryogenic Temperature (CT) on Tensile Properties and Fatigue Behavior of 2024-Al2O3 Nanocomposites”, Materials Science and Engineering, ICEMEA-2020, IOP Conf. Series, vol. 765, No.012052, 2020.
[30] A. D. Assi & H. O. Salman “The effect of tempering on the fatigue Behavior for medium carbon steel”, *Iraqi Journal for Mechanical and Material Engineering*, vol. 11, no. 4, 2011.
[31] E. Conrado, S. Foletti and C.Gorla, “A multi axaial fatigue approach for the calculation of the pitting strength of case hardened gears,” JSME International Conference on Motion and power Transmissions, vol. 11, pp. 356-362.
[32] A. D. Assi, “Study of Transverse and Longitudinal Crack Propagation in Human Bone using the Finite Element Method with MATLAB”, *Al-Khwarizmi Engineering Journal*, vol. 15, No. 2, 2019.
[33] Msc/Corporation. MSC/Fatigue Quick Start Guide, Vesion 8. Los Angeles: The MacNeal-Schwendler Corporation; 1999.
[34] E. Zahavi and V. Torbilo, “Fatigue design life expectancy of machine parts, 1st edition, CRC Press, vol. 321 pp.534, 1996.
[35] A. D. Assi, “Design Support System by MATLAB for Selection Steel Alloys,” *Journal of University of Babylon*, vol. 23, no. 1, 2015.